

Some Notes on the Value of Ranking Realizations

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It is relatively fast to generate multiple geostatistical realizations. Processing those realizations through a full process simulation is time consuming and expensive. It is becoming standard practice to rank the realizations by a quick-to-calculate statistic that is related to the full process modeling. Uncertainty can be assessed by using fewer realizations.

Setting of the Problem

We quantify the value of ranking by the reduction in the number of realizations required to assess uncertainty. A base case level of uncertainty is that obtained by 100 Monte Carlo Simulation (MCS) realizations. Clearly, uncertainty is improved if more realizations are processed; however, a measure of ranking goodness relative to the results of 100 MCS realizations.

$$N_R = \text{Number of Ranked Realizations} \equiv 100 \text{ MCS Realizations} \quad (1)$$

We would like NR as low as possible. The results of 100 MCS realizations will be summarized by the expected goodness of the predicted 10%, 50% and 90% quantiles, that is, the $z_{0.1}$, $z_{0.5}$ and $z_{0.9}$ values. We must consider the expected goodness because, by chance, one particular set of 100 realizations could be particularly good or particularly bad. The goodness of a set of probabilities will be judged relative to the true probabilities at the three quantiles we are interested in:

$$D = D(z_{0.1}, z_{0.5}, z_{0.9}) = E \left\{ \frac{|F(z_{0.1}) - 0.1| + |F(z_{0.5}) - 0.5| + |F(z_{0.9}) - 0.9|}{3} \right\} \quad (2)$$

where $z_{0.1}$, $z_{0.5}$ and $z_{0.9}$ are the estimated quantiles, D is a statistic very similar to the Kolmogorov-Smirnov test statistic used in classical statistics. Ideally, D would be as small as possible. We limit ourselves to the three quantiles because we cannot expect more from 100 realizations. The $F(z)$ values are the true probabilities associated to the predicted z quantiles. $F(z_{0.1}) \neq 0.1$ when we have a poor distribution.

The D value will decrease as the number of realizations increases. We must establish this relationship and the value of D for 100 realizations. The value of D does not depend on the shape of the distribution because the units are in the interval of 0 to 1 (probability units); we do not consider the actual Z units for this reason. To assess D for a given number of realizations L , we perform the following steps:

- Repeat a large number of times:
 1. Generate L random numbers uniformly distributed between 0 to 1: $p_l, l=1, \dots, L$
 2. Sort the p_l values in ascending order and extract the 0.1, 0.5 and 0.9 quantiles.
 3. Calculate the D statistic for this set of values. The true distribution is $U(0,1)$.

- Calculate the average D statistic over many realizations.

There should be a nice theoretical way to get the D value analytically, but we can calculate it numerically very easily. The results are shown on Figure 1. Virtually any two variables plot as a straight line on a log-log plot, but the relationship looks very stable. The equation for the fitted log-log red curve:

$$\log_{10}(D) = -0.51 - 0.51 \cdot \log_{10}(L) \quad (2)$$

The D value for 100 realizations is about 0.03. We can appreciate the meaning of 0.03 by looking at 10 randomly chosen sets of 100 MCS realizations, see Figure 2.

Ranking

Rather than processing realizations by MCS, we can target them by a quick-to-calculate ranking measure. To understand the idea, consider generating 10 times the number of realizations we need ($10 \cdot L$). We will still only process L realizations, but we generate ten times as many as we need and rank them according to the ranking measure. Then, we choose the L realizations equally spaced according to the ranking measure. Figure 3 shows an example with a distribution of a ranking measure and choosing five realizations. If the ranking measure was perfect, then the 0.1, 0.5 and 0.9 quantiles would be perfectly selected (D would be zero).

We often choose a ranking measure based on previous experience or an expert judgment; for example, we know that the connected oil-in-place should be highly correlated with the recoverable reserves from flow simulation. Figure 4 shows schematic cross plots of three ranking measures versus a true response. Practice has shown that we can easily get ranking measures that are correlated by more than 0.7 to the truth.

We can calculate the D values using ranked realizations instead of randomly chosen MCS realizations. The p_l values are calculated with the following instead of taken as random.

$$p_l = G \left(\rho \cdot G^{-1} \left(\frac{l + 0.5}{L} \right) + \sqrt{1 - \rho^2} \cdot G^{-1} (p_{random}) \right) \quad (3)$$

This is based on an assumption of bivariate Gaussian behavior between the ranking measure and the true response. The results for a ranking measure of 0.9 are shown on Figure 5. We see that 42 realizations are needed instead of 100 to reach a D value of 0.03.

Figure 6 shows the number of ranked realizations (see Equation 1) equivalent to 100 MCS realizations for different ranking measures correlated to the truth between 0 and 0.99. There is a benefit to the ranking when the ranking measure is correlated more than 0.8.

Conclusions

Ranking geostatistical realizations is a good idea. Low, median and high realizations can be chosen based on a quick statistical ranking measure. This reduces the number of realizations that must be considered in full process simulation.

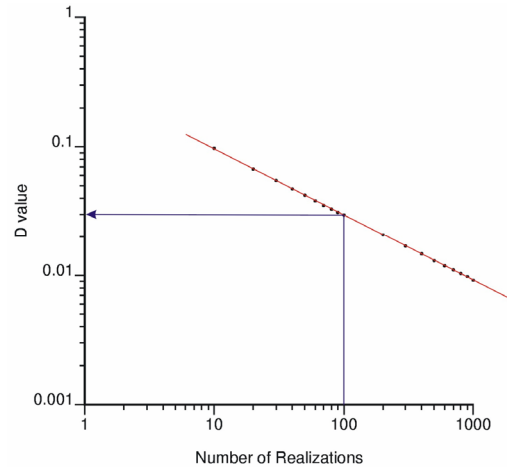


Figure 1: D value versus the number of realizations generated by Monte Carlo Simulation. The points are calculated numerically. The red line is fitted by regression. The blue line shows the results for 100 realizations.

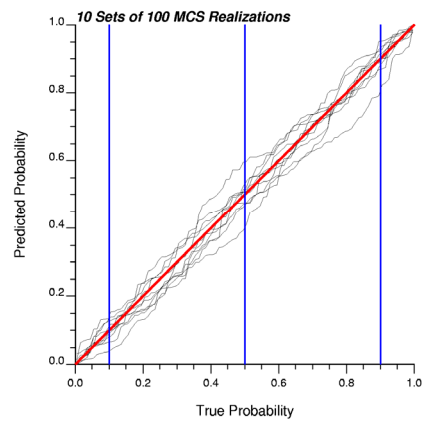


Figure 2: The results for 10 sets of 100 MCS realizations. The average absolute deviation at the 0.1, 0.5 and 0.9 true probability levels is 0.03 – each tic mark on the vertical axis is 0.025, which is just under the average absolute deviation of 0.03.

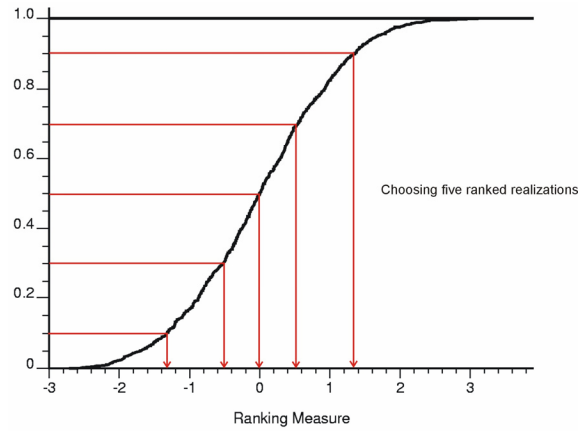


Figure 3: Cumulative distribution function of a ranking measure and choosing five ranked realizations. The realizations are chosen to uniformly sample the space of uncertainty.

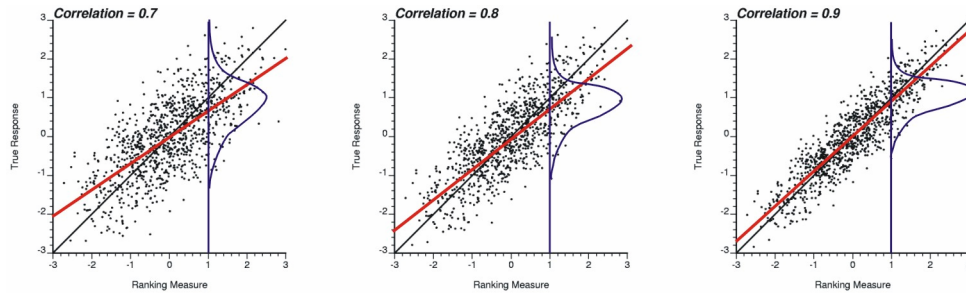


Figure 4: Schematic illustration of three ranking measures that are correlated with the true values with correlation coefficient values of 0.7, 0.8 and 0.9.

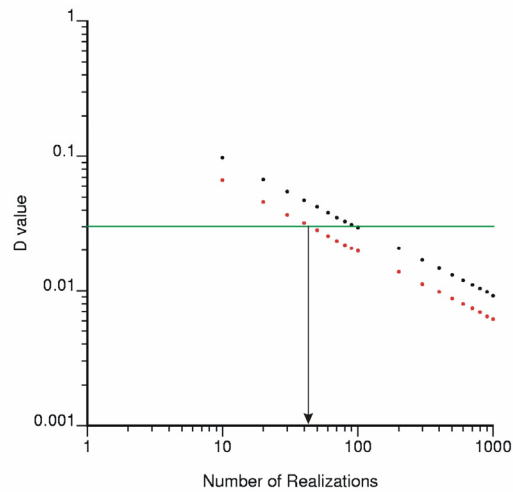


Figure 5: Improvement in the D values for a ranking measure correlated to the truth with 0.9. 42 realizations are needed instead of 100 to get to a measure of $D=0.03$.

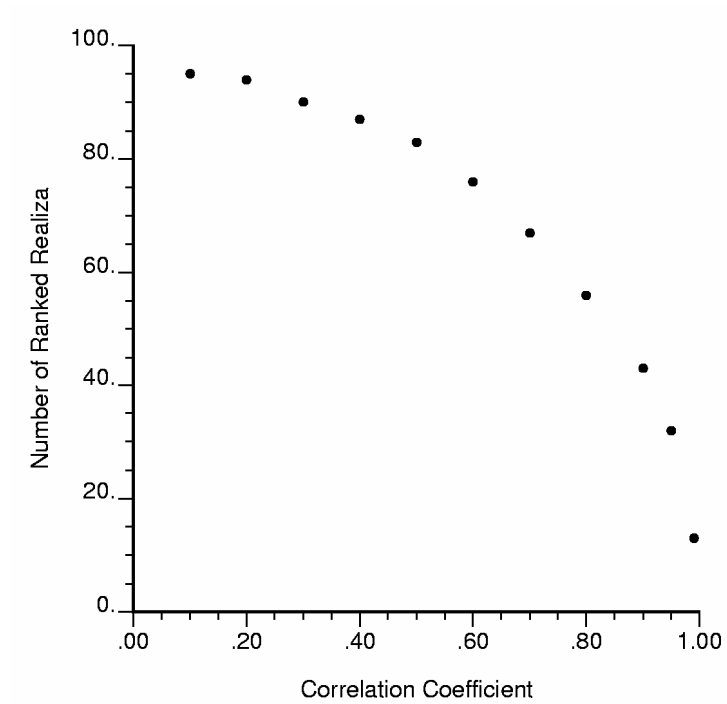


Figure 6: Improvement in the D values for different ranking measures correlated to the truth with between 0 and 0.99.